NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Procedural Fluency in Mathematics

A Position of the National Council of Teachers of Mathematics

Question

What is procedural fluency, and how do we help students develop it?

NCTM Position

Procedural fluency is a critical component of mathematical proficiency. Procedural fluency is the ability to apply procedures accurately, efficiently, and flexibly; to transfer procedures to different problems and contexts; to build or modify procedures from other procedures; and to recognize when one strategy or procedure is more appropriate to apply than another. To develop procedural fluency, students need experience in integrating concepts and procedures and building on familiar procedures as they create their own informal strategies and procedures. Students need opportunities to justify both informal strategies and commonly used procedures mathematically, to support and justify their choices of appropriate procedures, and to strengthen their understanding and skill through distributed practice.

Procedural fluency is more than memorizing facts or procedures, and it is more than understanding and being able to use one procedure for a given situation. Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem solving (NGA Center & CCSSO, 2010; NCTM, 2000, 2014). Research suggests that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999). Therefore, the development of students' conceptual understanding of procedures should precede and coincide with instruction on procedures. Although conceptual knowledge is an essential foundation, procedural knowledge is important in its own right. All students need to have a deep and flexible knowledge of a variety of procedures, along with an ability to make critical judgments about which procedures or strategies are appropriate for use in particular situations (NRC, 2001, 2005, 2012; Star, 2005).

In computation, procedural fluency supports students' analysis of their own and others' calculation methods, such as written procedures and mental methods for the four arithmetic operations, as well as their own and others' use of tools like calculators, computers, and manipulative materials (NRC, 2001). Procedural fluency extends students' computational fluency and applies in all strands of mathematics. For example, in algebra, students develop general equation-solving procedures that apply to classes of problems and select efficient procedures to use in solving specific problems. In geometry, procedural fluency might be evident in students' ability to apply and analyze a series of geometric transformations or in their ability to perform the steps in the measurement process accurately and efficiently.

Procedural fluency builds from an initial exploration and discussion of number concepts to using informal reasoning strategies and the properties of operations to develop general methods for solving problems (NCTM, 2014). Effective teaching practices provide experiences that help students to connect procedures with the underlying concepts and provide students with opportunities to rehearse or practice strategies and to justify their procedures. Practice should be brief, engaging, purposeful, and distributed (Rohrer,

2009). Too much practice too soon can be ineffective or lead to math anxiety (Isaacs & Carroll, 1999). Analyzing students' procedures often reveals insights and misunderstandings that help teachers in planning next steps in instruction. In the same way, worked examples can serve as a valuable instructional tool, permitting teachers to understand how students analyze why procedures work or don't work and consider what procedure might be most appropriate in a given situation (Booth, Lange, Koedinger, & Newton, 2013).

References and Additional Resources

Booth, J. L., Lange, K. E., Koedinger, K. R., & Newton, K. J. (2013). Using example problems to improve student learning in algebra: Differentiating between correct and incorrect examples. *Learning and Instruction, 25*, 24–34.

Hiebert, J. (1999). Relationships between research and the NCTM standards. *Journal for Research in Mathematics Education, 30*(1), 3–19.

Isaacs, A. C., & Carroll, W. M. (1999). Strategies for basic-facts instruction. *Teaching Children Mathematics, 5*(9), 508–515.

King, G., & Bay-Williams, J. M. (2014). Assessing basic fact fluency. *Teaching Children Mathematics, 20*(8), 488–497.

Lange, K. E., Booth, J. L., & Newton, K. J. (2014). Learning algebra from worked examples. *Mathematics Teacher, 107*(7), 535–540.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston VA: Author.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all.* Reston VA: Author.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common core state standards for mathematics. Common core state standards (college- and career-readiness standards and K–12 standards in English language arts and math).* Washington, DC: Author. http://www.corestandards.org.

National Research Council (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academies Press.

National Research Council. (2005). *How students learn: History, mathematics, and science in the classroom*. Washington, DC: National Academies Press.

National Research Council. (2012). *Education for life and work: Developing transferable knowledge and skills for the 21st century*. Washington, DC: National Academies Press.

Rohrer, D. (2009). The effects of spacing and mixed practice problems. *Journal for Research in Mathematics Education, 40*(1), 4–17.

Russell, S. J. (2000). Developing computational fluency with whole numbers. *Teaching Childdren Mathematics, 7*(3), 154–158.

Star, J. R. (2005). Reconceptualizing conceptual knowledge. *Journal for Research in Mathematics Education, 36*(5), 404–411.